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On Accurate Strong Non-Split Domination in Graphs

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Abstract. Accurate strong non-split dominating set in a graph with the relevant illustration is described in this manuscript. Minimal accurate strong non-split dominating set and accurate strong non-split domination numbers are specified for graphs. For some popular graphs, bounds on accurate strong non-split domination numbers are determined. Strong non-split domination number connections with other well-known dominating parameters are discovered. A few theorem relating to accurate strong non-split domination numbers are presented and obtained.

Keywords: Accurate non-split dominating set, Accurate strong non-split dominating set, Accurate non-split domination number, Accurate strong non-split domination number.

1 Introduction

Ore and Berge were the first to study dominating sets. V. R. Kulli et al brought the idea of Split domination[6], Non-split domination [7]. Strong split[8] and non-split domination[9] and Accurate dominating set[3]. K. Ameenal Bibi et al. introduced the notion of Accurate split and non-split domination[5]. In this paper, Accurate strong non-split dominating set and its domination numbers in graphs are stated and illustrated through examples.

2 Preliminaries

In this section, some basic definitions and theorems are given.

Definition 2.1: Let $G = (V, E)$ be a simple, finite connected and undirected graph. A set $D \subseteq V$ of a graph G is a dominating set(dom-set) if every vertex in $V - D$ is adjacent to

at least one vertex in D. The domination number(D-num) $\gamma(G)$ is the cardinality of a least dom-set of G.

Definition 2.2: A dom-set D of a graph $G = (V, E)$ is a non-split dom-set (NSD-set) if the induced subgraph $< V - D >$ is connected. The NSD-num $\gamma_{ns}(G)$ is the least cardinality of NSD-set.

Definition 2.3: A dom-set D of a graph $G = (V, E)$ is a strong NSD-set (SNSD-set) if the induced subgraph $< V - D >$ is complete. The $SNSD$-num $\gamma_{sns}(G)$ is the least cardinality of a SNSD-set.

Definition 2.4: A dom-set D of a graph $G = (V, E)$ is an accurate dom-set (AD-set) if V − D has no dom-set of cardinality |D|. The AD-num $\gamma_a(G)$ of G is the cardinality of a smallest AD-set of G.

Definition 2.5: A dom-set D of a graph $G = (V, E)$ is an accurate NSD-set ($ANSD$-set) if the induced sub graph $< V - D >$ is connected and it has no dom-set of cardinality $|D|$. The $ANSD$-num $\gamma_{ans}(G)$ of G is the least cardinality of an $ANSD$-set.

Theorem A[3]**:** For any complete graph, $\gamma_a(G) = \left\lfloor \frac{n}{2} \right\rfloor + 1, n \geq 3$.

Theorem B[9]**:** For every graph, $\alpha_0(G) \leq \gamma_{sns}(G)$ where $\alpha_0(G)$ is the independence number.

3 Accurate strong non-split domination in graphs

Accurate strong non-split dom-set and accurate non-split dom-num of a graph are introduced in this section

Definition 3.1: An $ANSD$-set $D \subset V$ in a graph G is stated to be an accurate $SNSD$-set($ASNSD - set$), if the induced subgraph $< V - D >$ is complete. ASNSD sets that are not proper subsets of any other ASNSD sets are known as minimal ASNSD sets. A minimum $ASNSD$-set is a $ASNSD$-set of minimum cardinalities in a graph. The $ASNSD$-num $\gamma_{asns}(G)$ is the least cardinality of an $ANSD$-set.

Example 3.2:

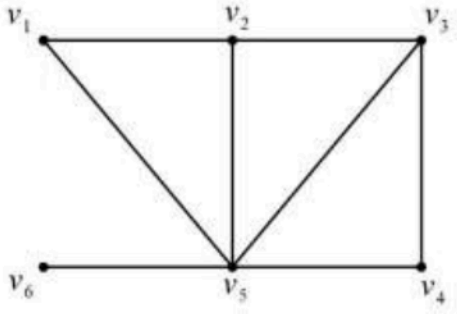

Fig 1. ASNSD-Graph

In this example, $\{v_5\}$ is a dom-set $\{v_5, v_6\}$ is a NSD −set. $\{v_3, v_4, v_6\}$ is a $SNSD$ −set. $\{v_1, v_2, v_3, v_4\}$ is an $ASNSD -$ set. Hence, $\gamma_{asns}(G) = 4$.

Observations:

1. For every connected graph, $n > 2$, $\gamma_{asns}(G) \leq n - 1$.
2. For any Tree, Since, it is acyclic, $< V - D >$ is either K_1 or K_2

Bounds on *ASNSD* number for a few notable graphs. In this section, bounds on *ASNSD* number for various graphs are determined.

Theorem 3.1: For every complete graph, $\gamma_{asns}(G) = \left\lfloor \frac{n}{2} \right\rfloor + 1, n \geq 3$.

Proof: By the theorem A, $\gamma_a(G) = \left\lfloor \frac{n}{2} \right\rfloor + 1$. By definition, *AD*- set is said to be *ASNSD*-set if $< V - D >$ is complete. Therefore, $\gamma_{asns}(G) = \left\lfloor \frac{n}{2} \right\rfloor + 1, n \geq 3$.

Theorem 3.2: For any path graph $P_n, n \geq 3$,

$$\gamma_{asns}(P_n) = \begin{cases} n-2, & n \geq 5 \\ n-1, & n = 3,4 \end{cases}$$

Proof: Let us labelling the vertices of a path as $v_1, v_2, v_3, \ldots, v_{n-1}, v_n$.
Case (i): If n=3,4, $\gamma_{asns}(P_n) = n - 1$.
Case (ii): If $n \geq 5$, For finding accurate strong non split dominating set D in any path, choose alternative vertices v_1, v_n, v_2, v_{n-1}, ... for D until to get $< V - D >$ is K_2.Therefore, $\gamma_{asns}(P_n) = n - 2$.

Theorem 3.3: For any cycle graph $C_n, n \geq 3$,

$$\gamma_{asns}(C_n) = \begin{cases} n-2, & n \geq 5 \\ n-1, & n = 3,4 \end{cases}$$

Proof: Let us labelling the vertices of a cycle as $v_1, v_2, v_3, \ldots, v_{n-1}, v_n$.
Case (i): If n=3,4, $\gamma_{asns}(C_n) = n - 1$.
Case (ii): If $n \geq 5$, For finding accurate strong non split dominating set D in any cycle, choose alternative vertices v_1, v_2, v_3, ... for D until to get $< V - D >$ is K_2.Therefore, $\gamma_{asns}(P_n) = n - 2$.

Theorem 3.4: For any wheel graph $W_n, n \geq 4$,

$$\gamma_{asns}(W_n) = \begin{cases} n-1, & n = 4 \\ n-2, & n = 5,6 \\ n-3, & n \geq 7 \end{cases}$$

Proof: Let $'u'$ be the central vertex adjacent to all vertices $v_1, v_2, \ldots, v_{n-1}$ of a cycle. Then $\{u\}$ alone a non split dominating but not accurate and strong and also for any cycle C_{n-1},

$$\gamma_{asns}(C_{n-1}) = \begin{cases} n-3, & n-1 \geq 5 \\ n-2, & n-1 = 3,4 \end{cases}$$

Case (i): If n=4, then ASNSD-set contains $\{u\}$ and $n - 2$ vertices. $\gamma_{asns}(W_n) = |\{u\}| + n - 2, \gamma_{asns}(W_n) = n - 1$.

Case (ii): If n=5,6, then ASNSD-set contains $\{u\}$ and $n-3$ vertices. $\gamma_{asns}(W_n) = |\{u\}| + n - 3$, $\gamma_{asns}(W_n) = n - 2$.
Case (iii): If $n \geq 7$, ASNSD-set does not contains $\{u\}$ since to leave $< V - D >$ as complete graph. Therefore, ASNSD-set contains $n-3$ vertices. $\gamma_{asns}(W_n) = n - 2$.

Theorem 3.5: For any star graph S_n or $K_{1,n-1}, n \geq 3$

$$\gamma_{asns}(S_n) = n - 1$$

Proof: Let $'u'$ be the internal node or central node of the graph. If $D = V - \{u\}$, then $< V - D >$ is K_1. Therefore D is the $ASNSD$-set of S_n. Therefore, $\gamma_{asns}(S_n) = n - 1$.

Theorem 3.6: For any complete bipartite graph $K_{m,n}, 2 < m \leq n$,

$$\gamma_{asns}(K_{m,n}) = m + n - 2, \qquad m \leq n$$

Proof: Since each vertex in V_1 is adjacent to every vertex in V_2 and each vertex among V_1 and among V_2 are not adjacent, choosing vertices from V_1 and V_2 alternatively till K_2 is obtained for D-set gives ASNSD-set. Therefore, ASNSD-set contains $m-1$ vertices and $n-1$ vertices. Thus $\gamma_{asns}(K_{m,n}) = m + n - 2$.

Relation between *ASNSD* number and other parameters. In this section, relation between $ASNSD$ number and other parameter for complete and comb graph are determined.

Theorem 3.7: For every connected graph G, $\gamma_{sns}(G) \leq \gamma_{asns}(G)$.
Proof: Every $ASNSD$-set is a $SNSD$-set. Hence the result.

Theorem 3.8: For every connected graph G,

$$\gamma_{ans}(G) \leq \gamma_{asns}(G)$$

Proof: It is obviously true.

Theorem 3.9: For any spanning subgraph SH of G, $\gamma_{asns}(G) \leq \gamma_{asns}(SH)$.
Proof: Since vertex degree reduced in spanning subgraph and every ASNSD-set of SH is a ASNSD-set of G, Hence the theorem holds.

Corollary 3.1: If G is either complete or cycle graph, then $\gamma_{asns}(G) = \gamma_{asns}(SH) - 1$

Theorem 3.10: For every graph G, $\alpha_0(G) \leq \gamma_{asns}(G)$ where $\alpha_0(G)$ is the independence number of G.
Proof: By theorem 3.7 and theorem B
We have $\alpha_0(G) \leq \gamma_{sns}(G) \leq \gamma_{asns}(G)$.

Theorem 3.11: Let T_c be a comb graph, then (i) $for\ n \geq 5$, $\gamma_{asns}(T_c) = n - 2$ and (ii) $for\ n = 4$, $\gamma_{asns}(T_c) = \gamma_{sns}(T_c) + 1$
Proof:

(i) Let $S = \{e_1, e_2, \dots e_r\}$ be the end vertices and $P = \{p_1, p_2, \dots p_r\}$ be the cut vertices where $|V| = |E| + |P|$. Since each vertex is adjacent to an end vertex, non-split dominating set (NSD-set) has $S = \{e_1, e_2, \dots e_r\}$ vertices but not accurate and strong non-split dominating set. Then adding a vertex from the set P, we get $ANSD$-set and hence $< V - D >$ is a path. In path, only K_2 exists. Therefore, $\gamma_{asns}(T_c) = n - 2$.

(ii) It is obviously true.

Corollary 3.2: For $n \geq 2$, $\gamma_{ns}(T_c) = \frac{n}{2}$

Proof: Removing all $\frac{n}{2}$ end vertices leaves the graph connected and hence the result.

Corollary 3.3: Let T_c be a comb graph, then $for\ \ n \geq 5$,

$$\gamma_{asns}(T_c) = \gamma_{sns}(T_c)$$

Proof: For both $ASNSD$-set and $SNSD$-set, $< V - D >$ is complete. In T_c, $< V - D >$ will be K_2. Hence the result.

Corollary 3.4: Let T_c be a comb graph, then $for\ \ n \geq 5$,

$$\gamma_{asns}(T_c) = \gamma_{sns}(T_c) = 2(\gamma_{ns}(T_c) - 1)$$

Proof: By theorem 3.11,

$$\begin{aligned}\gamma_{asns}(T_c) = \gamma_{sns}(T_c) &= n - 2 \\ &= 2\left(\frac{n}{2}\right) - 2 \\ &= 2\gamma_{ns}(T_c) - 2.\end{aligned}$$

Note: ω is a clique number and ω-set is a clique set.

Theorem 3.12: For every graph G, which is not complete and ≥ 5 , $\gamma_{asns}(G) \leq n - \omega(G) + 2$, where is a clique number of G.

Proof: Let V_ω be the collection of vertices of ω-set then $\forall\ l \epsilon V_\omega$, $(V - V_\omega) \cup \{l\}$ is a $SNSD$-set but may be accurate or may not be accurate. Hence add any other vertex $m \epsilon V_\omega$ then $(V - V_\omega) \cup \{l, m\}$ is an $ASNSD$-set. Therefore, $\gamma_{asns}(G) \leq |(V - V_\omega) \cup \{l, m\}|$. Hence, $\gamma_{asns}(G) \leq n - \omega(G) + 2$.

Corollary 3.5: For any graph G, which is not complete with $\omega(G) \geq \delta(G)$ and ≥ 5 , $\gamma_{asns}(G) \leq n - \delta(G) + 2$, where ω is a clique number of G and $\delta(G)$ is a minimum degree of G.

Nordhaus –Gaddum(NG) type results. Lower and upper bounds on $ASNSD$-set for graph G and its complement graph $\bar{G}$ without any isolated vertices are determined in the succeeding theorems,. NG type results are obtained.

Theorem 3.13: For every graph G and $\bar{G}$ without any isolated vertices,

$$2 \leq \gamma_{asns}(G)$$

$$2 \leq \gamma_{asns}(\bar{G})$$

Proof: Let G and $\bar{G}$ be a graph without any isolated vertices. Then there is no universal vertices in both G and $\bar{G}$. Therefore, $\gamma_{asns}(G) > 1$ and $\gamma_{asns}(\bar{G}) > 1$. That is, $\gamma_{asns}(G) \geq 2$ and $\gamma_{asns}(\bar{G}) \geq 2$.

Theorem 3.14: For every graph G and $\bar{G}$ withoutany isolated vertices,

$$\gamma_{asns}(G) \leq (n-1)$$
$$\gamma_{asns}(\bar{G}) \leq (n-1)$$

Proof: By definition of $ASNSD$-set, $V - D$ has at least K_2 or K_1. Therefore, $\gamma_{asns}(G)$ and $\gamma_{asns}(\bar{G})$ has at most $(n-1)$ vertices.

Theorem 3.15: For any graph G and $\bar{G}$ with no isolated vertices,

$$4 \leq \gamma_{asns}(G) + \gamma_{asns}(\bar{G}) \leq 2(n-1)$$
$$4 \leq \gamma_{asns}(G).\gamma_{asns}(\bar{G}) \leq (n-1)^2$$

Proof: By theorem 3.13 and 3.14,

$$2 \leq \gamma_{asns}(G) \leq (n-1)$$
$$2 \leq \gamma_{asns}(\bar{G}) \leq (n-1)$$

This implies,

$$4 \leq \gamma_{asns}(G) + \gamma_{asns}(\bar{G}) \leq 2(n-1)$$
$$4 \leq \gamma_{asns}(G).\gamma_{ass}n(\bar{G}) \leq (n-1)^2$$

4 Conclusion

The concepts of $ASNSD$-set in graphs are addressed in this study. We find many limits on $ASNSD$ numbers. In the future, applications and connections between $ASNSD$ numbers with different well known domination parameters might be developed.

References

1. V. R. Kulli, Advances in Domination Theory-I, Vishwa International Publications, Gulbarga, India (2012).
2. V. R. Kulli, Theory of Domination in Graphs, Vishwa International Publications, Gulbarga, India (2010).
3. V. R. Kulli and M.B. Kattimani, Accurate Domination in Graphs, Advances in Domination Theory-I, ed. V.R.Kulli, Vishwa International Publications, Gulbarga, India, 1-8 (2012).
4. V. R. Kulli and M.B. Kattimani, Connected Accurate Domination, J. Comput. math sci. 6 (2015) 682-687.
5. K. Ameenal Bibi, A. Lakshmi and R. Jothilakshmi, Accurate Split Domination, Accurate Non-Split Domination of a Graph, International Journal of Computational and Applied Mathematics, Research India Publications, Vol 12, No.1(2017), pp.115- 124.
6. V. R. Kulli and B. Janakiram, The Split domination number of a Graph, Graph Theory Notes of New York XXXII, 16-19, New York Academy of Sciences. (1997)
7. V. R. Kulli and B. Janakiram, The non-split domination number of a Graph, The Journal of Pure and Applied Math. Monthly 31(5). Pp 545-550(2000)

8. V. R. Kulli and B. Janakiram, The Strong Split domination number of a Graph, Gulbarga, India, Acta Ciencia Indica, Vol XXXII M, No.2, 715(2006)
9. V. R. Kulli and B. Janakiram, The Strong non-split domination number of a Graph Gulbarga, India, International journal of Management and Systems Vol. 19 No.2 may-august(2003).